MCP-003-001513 Seat No. ________

# B. Sc. (Sem. V) (CBCS) Examination

**May / June - 2018**

## Mathematics : Paper BSMT-501 (A)

***(Theory) (Mathematical Analysis & Group Theory)***

**Faculty Code : 003**

**Subject Code : 001513**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

**1** Answer the following questions : **20**

(1) Give an example of group which not abelian.

(2) Define Automorphism.

(3) Define Normal Subgroup.

(4) Define Order of Group

(5) Find generators of Cyclic Group $(Z_8, +_8)$

(6) Define Right Coset.

(7) Define Isomorphism of Group.

(8) Define Factor Group.

(9) Examine whether the following permutation is even or odd

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 5 & 2 & 6 & 8 & 1 & 7 \end{pmatrix}$$

(10) Find 0(f) where $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 2 & 9 & 4 & 6 & 1 & 3 & 8 & 5 \end{pmatrix}$

(11) If $f(x) = \frac{20}{x}$, $x \in [2, 20]$ and P = { 2, 4, 5, 20 } be a partition then $\|P\|$ is ________.

(12) $\int_{-1}^{1} |x| \, dx =$ ________.

(13) State Darboux's Theorem.

(14) Define Neighborhood of a point.

(15) Define : Separable metric space.

(16) If E = (1,2) is a subset of metric space R then E' = ___________.

(17) If (X, d) is discrete metric space and $1 < \delta$ then $N(a, \delta)$ = ___________.

(18) Define: Limit point.

(19) Define : Isolated point.

(20) Define : Countable Set.

**2** (A) Attempt any three **6**

(1) Define Upper Riemann Integration and Lower Riemann Integration.

(2) If function f is continuous in [a, b] then $f \in R_{[a,b]}$.

(3) Prove that $\frac{1}{2} < \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^{2n}}} < \frac{\pi}{6}$.

(4) If (X, d) is a metric space and $A \subset B \subset X$ then prove that $A^\circ \subset B^\circ$.

(5) If $E_n = \left(\frac{1}{n}, \frac{n-1}{n}\right)$ then check subset $\bigcap_{n=3}^{\infty} E_n$ of metric space $R$ is open or closed.

(6) Prove that every finite subset of metric space is closed.

(B) Attempt any three : **9**

(1) State and prove first mean value theorem of integral calculus.

(2) Prove that $\lim_{n \to \infty} \left(\frac{n^n}{n!}\right)^{\frac{1}{n}} = e$.

(3) If $f \in R_{[a,b]}$ then for $\lambda > 0, \lambda f \in R_{[a,b]}$.

(4) State and prove Housedroff's Principle for a metric space.

(5) Prove that $\frac{1}{4}$ is in cantor set.

(6) If $A = \left\{\frac{1}{n} + \frac{1}{m} / n, m \in N\right\} \subset R$ then find A'

(C) Attempt any two : **10**

(1) Let f be bounded function on [a, b] then necessary and sufficient condition for

$$\int_a^b f(x)\,dx = \overline{\int_a^b} f(x)\,dx = \underline{\int_a^b} f(x)\,dx \text{ is } \lim_{\|P\|\to 0} S(P, f) \text{ exist}$$

and value of this limit is $\int_a^b f(x)\,dx$.

(2) Let f be bounded function on $[a,b]$. P and P* are two partition of $[a,b]$ such that $P \subset P^*$ then $L(P,f) \le L(P^*,f) \le U(P^*,f) \le U(P,f)$.

(3) Prove that (R, d) is separable metric space.

(4) Prove that any subset of discrete metric space is both open and closed.

(5) If $(X,d)$ is metric space then prove that $\left(X, \dfrac{d}{1+d}\right)$ is also a metric space.

3 (A) Attempt any three **6**

(1) Let (G, *) be a group then prove that $(a*b)^{-1} = b^{-1} * a^{-1},\ \forall a, b \in G$.

(2) If Binary operation * defined as $a*b = ab+1, \forall\, a, b \in G,$ Is $(G,*)$ group or not ?

(3) Let G be group and a, b $\in$ G such that a # e and O(b) = 2 if $bab^{-1} = a^2$, then find O(a).

(4) Let $H \le G$ and $a, b \in G$, if $H_a = H_b$, then prove that $ab^{-1} \in H$.

(5) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 3 & 1 & 2 & 4 \end{pmatrix}$ and

$g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 2 & 4 & 6 & 1 \end{pmatrix}$, then find $f_0 g$ and $g_0 f$.

(6) Prove that every cyclic group is abelian.

(B) Attempt any three : **9**

(1) Using Fermat's theorem, find the remainder when $3^{256}$ is divided by 14.

(2) If $H \le G$ then show that $x^{-1}Hx = \left\{x^{-1}hx / h \in H\right\}$ is also subgroup of $G; \forall x \in G$.

(3) Prove that composition of two disjoint cycles in $S_n$ is commutative.

(4) Show that a non empty subset H of group G is subgroup of G iff $ab^{-1} \in H, \forall a, b \in H$.

(5) A subgroup H of G is Normal iff $x^{-1}hx \in H, \forall x \in G, \forall h \in H$.

(6) Prove that the set of function $\left\{f_1, f_2, f_3, f_4\right\}$ on C define by $f_1(z) = z, f_2(z) = -z$, $f_3(z) = \frac{1}{z}, f_4(z) = \frac{-1}{z}$ forms an abelian group under binary operation composition.

(C) Attempt any two **10**

(1) State and prove Lagrange's Theorem.

(2) State and prove Cayley's Theorem.

(3) Define Alternating Group $A_n$, Show that $A_n (n \ge 2)$ is a subgroup of $S_n$ of order $\frac{n!}{2}$.

(4) Prove that Isomorphism of group is an equivalent relation.

(5) A subgroup H of group G is normal subgroup iff $(H_a)(H_b) = H_{ab}$ ; $\forall a, b \in G$.